

# Riding the Tether to the Moon: A Multi-Objective Optimization Framework for Hybrid Logistics Across Operational Scenarios

## Summary

This paper addresses the logistics optimization for constructing a 100,000-person lunar colony after 2050, developing a **unified model framework** enabling multi-scenario analysis through parameter adjustment.

First, we incorporate all relevant factors at the modeling stage—including tether sway effects on transport efficiency and equipment failure downtime—so that subsequent scenario analysis requires only parameter changes. We construct a **three-tier architecture**: Sub-model 1 and Sub-model 2 characterize the time-cost properties of the **Space Elevator System (SES)** and **Traditional Rocket Launch (TRL)**, corresponding to Schemes A and B; the integrated model uses the allocation ratio as the decision variable, representing Scheme C.

Subsequently, we divide the project timeline into the **construction phase** and **sustaining phase** with different optimization frameworks. During construction, the objective is minimizing total cost and time for transporting 100 million metric tons, with environmental impact as a constraint. During sustaining, the objective shifts to minimizing environmental impact while meeting water demand for 100,000 residents, with cost and reliability as constraints. This objective switching enables phase-specific recommendations.

Furthermore, for all three models across both phases, we examine ideal conditions (no sway, no failures) and non-ideal conditions. Algorithmically: **NSGA-II** solves the Pareto frontier for cost, time, and environmental impact; **Monte Carlo simulation** evaluates uncertainty propagation from stochastic parameters under non-ideal conditions; **CVaR robust optimization** controls extreme scenario losses through weighted combination of expected impact and tail risk; **TOPSIS** selects compromise solutions from the Pareto frontier.

Results show: under ideal construction conditions, Scheme A requires **186.2 years** at **\$5.69 trillion** with zero environmental impact; Scheme B requires **124.0 years** at **\$52.94 trillion** with severe impact; Scheme C (**77% SES allocation**) achieves Pareto optimality at **146.2 years** and **\$15.96 trillion**. Under non-ideal conditions, Scheme B's time increases by **171.6%**, while Scheme A increases only **21.8%**, confirming SES robustness. During sustaining, a critical water recycling threshold of **95.3%** is identified: above this, SES alone meets demand with zero impact; below, rocket supplementation is required. CVaR optimization achieves **99.1% supply reliability**.

Finally, sensitivity analysis validates model robustness: SES allocation dominates construction-phase costs; water recycling efficiency dominates sustaining-phase outputs. We recommend an **SES-dominant hybrid strategy ( $\geq 80\%$  allocation)** with recycling efficiency above 95%.

**Keywords:** Space Elevator Logistics Optimization; NSGA-II Multi-objective Optimization; CVaR Robust Optimization; Monte Carlo Simulation

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# 1 Introduction

## 1.1 Problem Background

As Earth's resources become increasingly strained and the global population continues to grow, humanity's expansion into outer space has evolved from a science fiction vision into a tangible agenda. The Moon, being the closest celestial body to Earth, has emerged as the premier candidate for establishing an extraterrestrial colony, owing to its abundant helium-3 reserves and low-gravity environment.

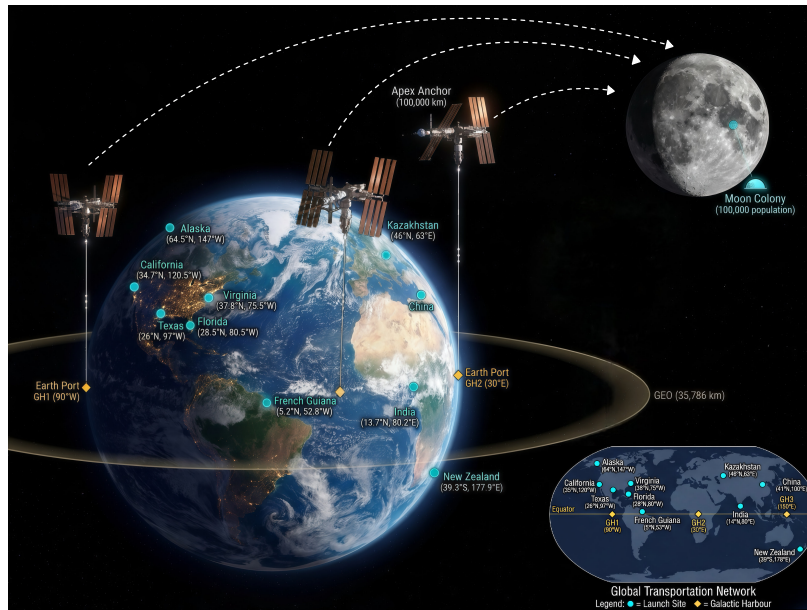


Figure 1: Space Elevator System Schematic Diagram

However, conventional chemical rockets impose prohibitive costs and substantial carbon emissions, severely limiting the feasibility of large-scale space logistics. Meanwhile, the emerging Space Elevator technology, despite its advantages of low energy consumption and reusability, still presents uncertainties regarding payload capacity and construction timelines. Consequently, determining the optimal balance among cost, efficiency, and environmental impact to develop a scientifically sound transportation strategy for lunar colonization has become a critical decision-making challenge as humanity advances toward an interplanetary civilization.

## 1.2 Restatement of the Problem

According to the problem requirements, this study needs to address the following core issues:

- **Problem 1:** For the transportation demand of 100 million metric tons of construction materials, develop mathematical models to compare the time and cost characteristics of three transportation schemes: Scheme A (using only the Space Elevator System's three Galactic Harbours), Scheme B (using only traditional rocket launches), and Scheme C (a combination of the Space Elevator and traditional rockets).
- **Problem 2:** Under non-ideal operating conditions (tether swaying, rocket failures, elevator breakdowns, etc.), analyze the degree of change and robustness of model results.

- **Problem 3:** Investigate the annual water resource requirements for the 100,000-person population after the colony is fully operational, and evaluate the additional cost and timeline needed for the sustaining phase.
- **Problem 4:** Quantify the environmental impact on Earth under different scenarios, and explore optimization strategies to minimize environmental damage.
- **Problem 5:** Submit a one-page recommendation letter to the MCM Agency, comprehensively considering the above factors to recommend an optimal course of action.

### 1.3 Our work

In order to clearly illustrate our work, we draw the flowchart Figure 2.

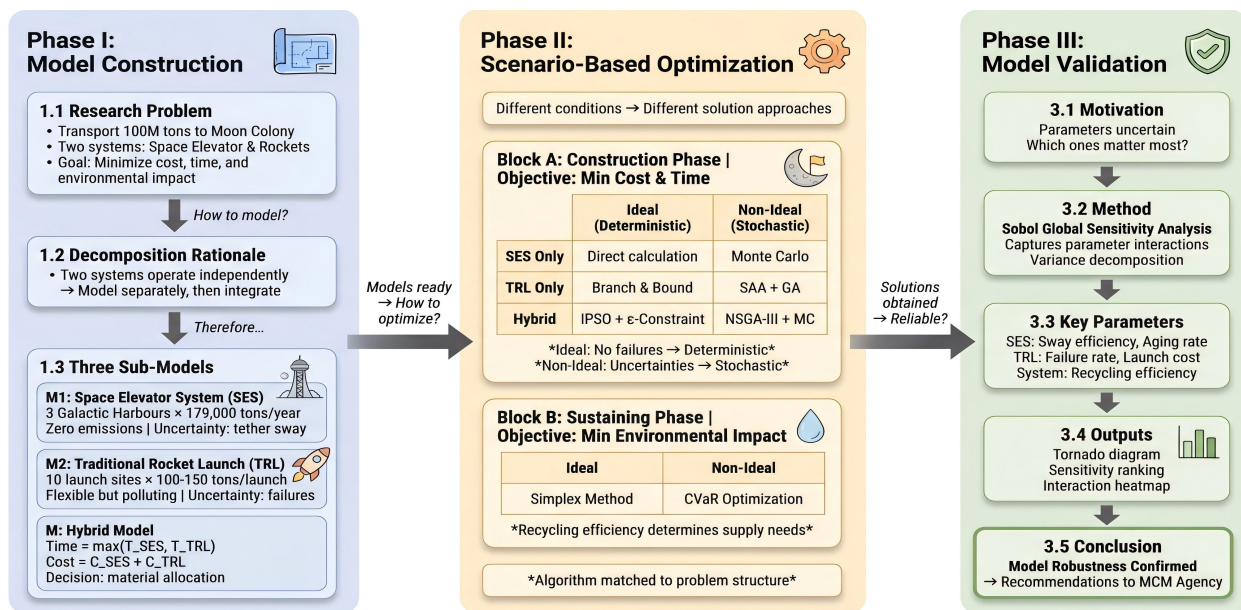


Figure 2: Our work

## 2 Assumptions and Justifications

To ensure the operability of the model and the reasonableness of the results, we propose the following assumptions based on problem information and engineering practice:

- **Assumption 1:** The Moon Colony project can be divided into a Construction Phase and a Sustaining Phase, with the logistics demands, constraint conditions, and optimization objectives of the two phases being mutually independent and modeled separately.

**Justification:** The Construction Phase focuses on large-scale material transportation under time and cost constraints; the Sustaining Phase focuses on small-batch, high-frequency replenishment pursuing long-term stability. Separate modeling aligns with phased management philosophy in engineering practice.

- **Assumption 2:** The SES produces no atmospheric pollution during operation, and environmental impacts come solely from traditional rocket launches.

**Justification:** The problem explicitly states that SES is “generating no atmospheric pollution,” with electricity as its power source involving no combustion of chemical propellants.

- **Assumption 3:** Throughout the entire project cycle, the cost reduction effect brought by technological progress can be described using a constant learning rate  $r_{learn}$ , meaning that for each doubling of cumulative production, unit cost decreases by a fixed proportion.

**Justification:** Cost reduction in the aerospace industry typically follows Wright’s Law, validated by SpaceX’s reusable rocket cost trends.

- **Assumption 4:** The water recycling system efficiency  $\eta_{recycle}$  of the Moon Colony can reach the range of 90%-98%, consistent with the engineering goals of closed-loop life support systems.

**Justification:** The ISS water recycling efficiency has reached 93%, and future lunar bases are expected to achieve higher levels.

### 3 Notations

For ease of model presentation, the main symbols and their definitions used in this paper are shown in Table 1.

Table 1: Notation

Symbol	Description	Unit	Symbol	Description	Unit
<i>Basic Parameters</i>					
$M_{total}$	Total material demand	tons	$T_{total}$	Total completion time	yr
$M_{SES}$	SES transport volume	tons	$C_{total}$	Total cost	\$
$M_{TRL}$	TRL transport volume	tons	$r_{mass}$	Mass coefficient	-
<i>Space Elevator System (SES) Parameters</i>					
$N_{harbor}$	Number of Galactic Harbours	-	$C_{SES}$	SES total cost	\$
$R_{single}$	Capacity per Harbour	t/yr	$c_{SES}^{unit}$	SES unit transport cost	\$/kg
$R_{SES}(t)$	SES transport rate	t/yr	$c_{climb}$	Tether climbing cost	\$/kg
$\eta(t)$	Swaying efficiency factor	-	$c_{transfer}$	Orbital transfer cost	\$/kg
$A_{sys}(t)$	System availability	-	$c_{ops}$	Operational cost	\$/kg
$\lambda$	Failure intensity	1/yr	$C_{annual}^{fixed}$	Fixed maintenance cost	\$/yr
$\tau$	Mean time to repair	yr	$r_{learn}$	Learning rate	-
$T_{SES}$	SES completion time	yr	$C_{base}$	Aging maintenance cost	\$/yr
			$\beta$	Aging decay coefficient	1/yr
			$C_{incident}^{per}$	Cost per incident	\$
<i>Traditional Rocket Launch (TRL) Parameters</i>					
$N_{sites}$	Number of launch sites	-	$N_j$	Launches at site $j$	-
$J_{active}$	Selected sites set	-	$t_{inv}$	Investigation time	yr

*Continued on next page*

Symbol	Description	Unit	Symbol	Description	Unit
$m_{payload}$	Payload per launch	tons	$T_{TRL}$	TRL completion time	yr
$p_j$	Failure rate of site $j$	-	$C_{TRL}$	TRL total cost	\$
$\bar{p}$	Average failure rate	-	$c_{launch,j}$	Launch cost at site $j$	\$
$f_{max,j}$	Max frequency of site $j$	/yr	$c_{payload}^{loss}$	Payload loss cost	\$
$P_{env,j}$	Env. adaptation prob.	-	$c_{infra}$	Infrastructure cost	\$/yr
$N_{real}$	Actual launches required	-			
<i>Environmental Impact Parameters</i>					
$E(t)$	Annual emission rate	t/yr	$ODI_{limit}$	Ozone Depletion limit	-
$L(t)$	Pollution load	tons	$RFI_{baseline}$	RF Interference baseline	-
$\alpha_{decay}$	Pollutant decay coeff.	1/yr	$EIS$	Env. Impact Score	-
$L_{peak}$	Peak pollution load	tons	$\alpha_{ozone}$	Ozone impact weight	-
			$\alpha_{RF}$	RF impact weight	-
<i>Sustaining Phase Parameters</i>					
$D_{water}$	Annual water demand	t/yr	$\Omega$	SES water quota limit	t/yr
$d_{pp}$	Per capita consumption	L/p/d	$x_{water}$	SES water volume	t/yr
$N_{pop}$	Colony population	-	$y_{water}$	TRL water volume	t/yr
$\eta_{recycle}$	Recycling efficiency	-			

## 4 Multi-Objective Optimization Model for a Lunar Colony

### 4.1 Overview of the Modeling Framework

Through an in-depth analysis of the problem, we identify the following key elements:

**(1) Transportation Mode Characteristics:** The SES features an annual capacity of 179,000 tonnes per Galactic Harbor and zero atmospheric emissions, with operating costs primarily comprising electricity consumption and maintenance expenses. TRL offers the advantages of technological maturity and decision-making flexibility, but faces high launch costs and environmental impacts. The two modes exhibit fundamental differences in cost structure, time characteristics, and environmental impact, and this complementarity provides room for optimizing a hybrid transportation scheme.

**(2) Decision Objectives:** The problem requires a comparison of the time and cost characteristics of three schemes while accounting for environmental impact. This naturally constitutes a time–cost–environment multi-objective optimization problem.

**(3) Constraints:** These include total transportation demand, system capacity limits, time constraints, budget limits, and environmental impact thresholds. Among these, some are inherent constraints determined by physical laws, while others are scenario constraints set by decision-makers.

#### 4.1.1 Modeling Principles

Based on the above problem analysis, we establish the following three core modeling principles to ensure the completeness, flexibility, and interpretability of the model:

##### (1) Completeness Principle

All models are presented in their complete form, fully accounting for real-world factors such as efficiency losses, failure risks, and aging effects. Different scenarios (e.g., ideal versus non-ideal conditions) are realized through parameter settings. The advantage of this principle is that it avoids redundant modeling for different scenarios and ensures the unity of the model framework.

### (2) Temporal Phasing Principle

The project lifecycle is divided into a Construction Phase and a Sustaining Phase. The Construction Phase focuses on the transportation of 100 million tonnes of building materials, aiming to complete the required transport volume within time and cost constraints. The Sustaining Phase addresses the long-term water and supply replenishment for a population of 100,000, prioritizing the stability and cost-effectiveness of sustained delivery. The models in this section are primarily constructed for the Construction Phase; the Sustaining Phase model will be developed as an extension in subsequent sections.

### (3) Constraint Layering Principle

A distinction is made between Inherent Constraints and Scenario Constraints. Inherent constraints originate from physical laws and system design—such as capacity limits and non-negativity constraints—and hold for all scenarios. Scenario constraints are determined by specific mission objectives—such as time constraints and budget constraints—and can be adjusted according to decision-making needs.

#### 4.1.2 Model Architecture

Based on the above principles, we construct a hierarchical logistics optimization model framework for the lunar colony. This framework comprises two sub-models and one integrated model:

- **Sub-model I—Space Elevator System Model (SES):** Characterizes the time–cost profile of the SES, providing a quantitative basis for Scheme A.
- **Sub-model II—Traditional Rocket Launch Model (TRL):** Characterizes the time–cost profile of TRL, providing a quantitative basis for Scheme B.
- **Integrated Model—Hybrid Transportation System Optimization Model:** Through the optimal allocation of decision variables  $M_{SES}$  and  $M_{TRL}$ , this model integrates the two sub-models to obtain Pareto-optimal solutions for Scheme C.

## 4.2 Sub-model I: Space Elevator System Model

### 4.2.1 Modeling Approach

The SES transportation process consists of two stages (as illustrated in Figure 3): first, an electrically powered climber transports cargo from the Earth port along a graphene tether to the apex anchor point; then, a low-fuel rocket completes the orbital transfer from the apex anchor point to the lunar colony. According to the problem setting, a complete SES consists of three Galactic Harbors ( $N_{harbor} = 3$ ), each with a design annual capacity of  $R_{single} = 179,000$  tonnes.

In constructing the SES model, we need to answer the following core questions: given the transport volume  $M_{SES}$  assigned to the SES, how long does it take to complete the transportation? What is the total cost? To this end, we develop a time model and a cost model separately.

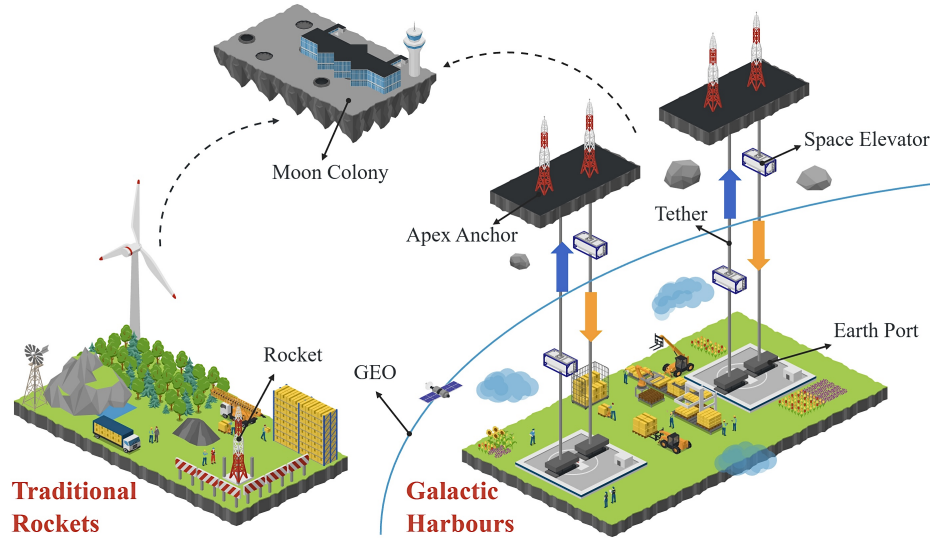


Figure 3: Space Elevator System Schematic Diagram

#### 4.2.2 SES Time Model

For the time model, the key lies in determining the effective transport rate of the SES. In practice, however, two types of efficiency losses typically arise: first, capacity reduction due to tether oscillation, captured by the oscillation efficiency factor  $\eta(t)$ ; second, downtime due to equipment failures, captured by the system availability  $A_{sys}(t)$ . By introducing these two factors, the model can handle both ideal and non-ideal operating conditions within a unified framework.

##### (1) Modeling the Effective Transport Rate

The effective transport rate of the SES depends on both the designed system capacity and the actual operating conditions. Following the performance degradation principle proposed by Rausand and Hoyland [1], we introduce efficiency factors to derate the system capacity. The effective annual transport rate  $R_{SES}(t)$  of the SES at time  $t$  is defined as:

$$R_{SES}(t) = N_{harbor} \cdot R_{single} \cdot \eta(t) \cdot A_{sys}(t) \quad (1)$$

The base capacity  $N_{harbor} \cdot R_{single}$  represents the designed system capability under ideal conditions, while  $\eta(t)$  and  $A_{sys}(t)$  derate the base capacity from the perspectives of operational efficiency and equipment availability, respectively. This multiplicative performance modeling approach is discussed in detail in Birolini's [2] work on reliability engineering.

##### (2) Modeling System Availability

$A_{sys}(t)$  reflects the impact of equipment failures on operating time. Following the failure–repair process availability theory systematically presented by Birolini, we adopt the steady-state availability formula. For exponentially distributed failure inter-arrival times and repair times, the steady-state availability can be expressed as:

$$A_{sys}(t) = \frac{1}{1 + \lambda(t) \cdot \tau(t)} \quad (2)$$

The physical meaning of this expression can be understood through the "busy period–idle period" queuing model described by Rausand and Hoyland:  $\lambda \cdot \tau$  represents the average fraction of

downtime per unit time, so  $A_{sys}$  gives the proportion of time during which the system is operational. This formula assumes that the system is in a steady-state operating phase, which is consistent with the separation of the Construction Phase and Sustaining Phase proposed in our assumptions.

### (3) Completion Time Model

Given the total transport volume  $M_{SES}$  assigned to the SES, the time  $T_{SES}$  required to complete the transportation must satisfy the following integral constraint on transport volume:

$$\int_0^{T_{SES}} R_{SES}(t) dt = M_{SES} \cdot r_{mass} \quad (3)$$

In a simplified analysis, if  $R_{SES}(t)$  is assumed to be a constant  $R_{SES}$ , the completion time can be expressed in closed form as:

$$T_{SES} = \frac{M_{SES} \cdot r_{mass}}{R_{SES}} = \frac{M_{SES} \cdot r_{mass}}{N_{harbor} \cdot R_{single} \cdot \eta \cdot A_{sys}} \quad (4)$$

### 4.2.3 SES Cost Model

For the cost model, following the life-cycle cost analysis framework proposed by Birolini, we decompose the SES operating costs into four categories: variable costs  $C_{SES}^{var}$  proportional to transport volume, fixed maintenance costs  $C_{SES}^{fixed}$  related to operating time, aging maintenance costs  $C_{SES}^{aging}$  that grow exponentially over time, and incident costs  $C_{SES}^{incident}$  based on a stochastic process.

#### (1) Cost Structure Analysis

The SES operating costs comprise four components. **Variable costs** scale linearly with transport volume (electricity, propellant, handling). **Fixed maintenance costs** incorporate a learning rate factor  $(1 - r_{learn})^t$  following Wright's learning curve model [3]. **Aging costs** capture tether material degradation under extreme space conditions via an exponential function  $e^{\beta t}$  [2]. **Incident costs** follow a Poisson process with expected incidents  $\lambda \cdot T_{SES}$  [1].

Combining the above analysis, the total cost model for the SES is:

$$C_{SES} = \underbrace{M_{SES} \cdot c_{SES}^{unit}}_{\text{Variable cost}} + \underbrace{\int_0^{T_{SES}} C_{annual}^{fixed} \cdot (1 - r_{learn})^t dt}_{\text{Fixed maintenance cost}} + \underbrace{\int_0^{T_{SES}} C_{base}^{aging} \cdot e^{\beta t} dt}_{\text{Aging cost}} + \underbrace{\lambda \cdot T_{SES} \cdot C_{per}^{incident}}_{\text{Incident cost}} \quad (5)$$

### 4.2.4 SES Constraints

Following the constraint layering principle, we classify the constraints into two categories.

#### (1) Inherent Constraints (valid for all scenarios):

- **Capacity limit constraint:** The actual annual transport volume of the SES cannot exceed the design limit of the system, i.e., the total annual capacity of the three Galactic Harbors:

$$\frac{dM_{SES}(t)}{dt} \leq N_{harbor} \cdot R_{single} \quad (6)$$

- **Non-negativity constraint:** Transport volume and completion time are both non-negative physical quantities:

$$M_{SES} \geq 0, \quad T_{SES} \geq 0 \quad (7)$$

- **Efficiency bound constraint:** The oscillation efficiency factor and system availability are both proportional coefficients whose ranges are determined by definition.  $\eta(t) = 1$  indicates no oscillation loss, and  $A_{sys}(t) = 1$  indicates no failure-induced downtime:

$$0 < \eta(t) \leq 1, \quad 0 < A_{sys}(t) \leq 1 \quad (8)$$

(2) **Scenario Constraints** (set according to specific missions):

- **Time deadline constraint:** When the mission has a schedule requirement, the SES completion time must not exceed the specified deadline:

$$T_{SES} \leq T_{deadline} \quad (9)$$

- **Budget constraint:** When the mission has a budget limit, the total SES cost must not exceed the allocated budget:

$$C_{SES} \leq B_{SES} \quad (10)$$

This layered constraint design provides the model with good extensibility: inherent constraints guarantee the physical feasibility of solutions, while scenario constraints can be flexibly configured for different problem settings.

### 4.3 Sub-model II: Traditional Rocket Launch Model

#### 4.3.1 Modeling Approach

The TRL scheme transports cargo directly from launch sites on Earth to the lunar colony, representing a single-step, direct transportation mode. Unlike the two-stage SES mode, the TRL transportation process is more straightforward; however, each launch consumes a large amount of chemical propellant, resulting in high costs and significant atmospheric emissions.

According to the problem information, there are  $N_{sites} = 10$  available launch sites worldwide, with a rocket payload capacity of  $m_{payload} \in [100, 150]$  tonnes. In constructing the TRL model, the core questions are: given the transport volume  $M_{TRL}$  assigned to TRL, how many launches are required? How long do these launches take to complete? What is the total cost?

#### 4.3.2 TRL Time Model

Unlike the SES, the TRL time model must account for the impact of launch failures. Following the system reliability analysis method proposed by Rausand and Hoyland, we model the launch process as a Bernoulli trial sequence: each launch is conducted independently with a success probability of  $(1 - p_j)$  and a failure probability of  $p_j$ . A launch failure not only results in the loss of the payload but also triggers an accident investigation, causing time delays. This stochastic process is conceptually depicted in Figure 4.

##### (1) Computing the Effective Number of Launches

Considering launch failure risk, we need to calculate the actual number of launches required to complete the given transport volume. Following the redundancy system analysis method described

by Birolini in reliability engineering, when a system is subject to random failures, the expected number of attempts required to complete a task is inversely proportional to the success rate. Accordingly, the actual number of launches  $N_{real}$  required to complete the transport volume  $M_{TRL}$  is:

$$N_{real} = \frac{M_{TRL} \cdot r_{mass}}{m_{payload} \cdot (1 - \bar{p})} \quad (11)$$

The meaning of this formula is as follows: given an average failure rate of  $\bar{p}$ , the expected effective payload per launch is  $m_{payload} \cdot (1 - \bar{p})$ ; therefore, fulfilling the transport demand of  $M_{TRL} \cdot r_{mass}$  requires  $N_{real}$  launches.

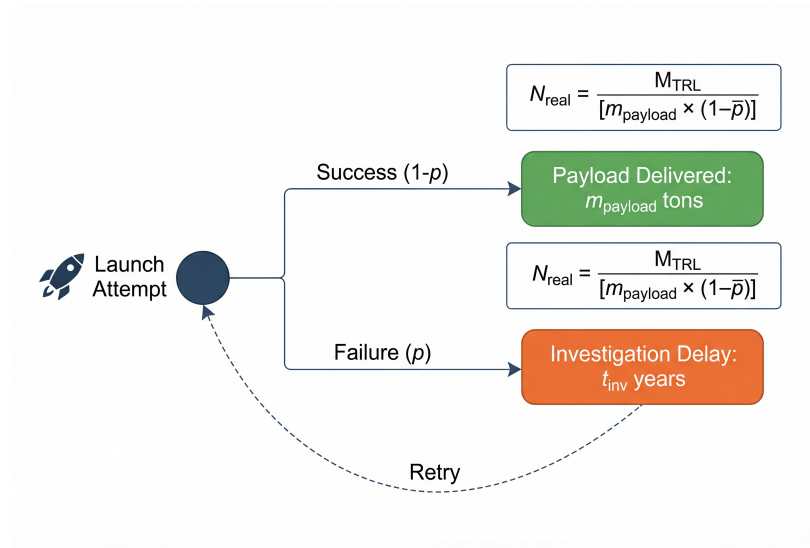


Figure 4: TRL Launch Process Bernoulli Decision Tree.

## (2) Completion Time Model

The total TRL completion time consists of two components: the time required for nominal launches and the investigation delay time following failures. The TRL completion time model is:

$$T_{TRL} = \underbrace{\frac{N_{real}}{\sum_{j \in J_{active}} f_{max,j} \cdot P_{env,j}}}_{\text{Nominal launch time}} + \underbrace{\frac{N_{real} \cdot \bar{p} \cdot t_{inv}}{N_{sites}}}_{\text{Failure investigation delay}} \quad (12)$$

The first term represents the nominal launch time: the denominator is the effective annual launch capacity of the system (the sum of actual launch frequencies adjusted for the environmental suitability probability). The second term represents the failure investigation delay:  $N_{real} \cdot \bar{p}$  is the expected number of failures, and it is assumed that investigations can be conducted in parallel across  $N_{sites}$  launch sites, so the total delay is distributed over the number of sites.

### 4.3.3 TRL Cost Model

Following the life-cycle cost framework proposed by Birolini, we decompose TRL costs into three components: launch costs directly related to the number of launches, payload loss costs due to failures, and infrastructure maintenance costs related to operating time. Jones notes that, with

technological advances by commercial space companies such as SpaceX, rocket launch costs have been reduced by approximately 20-fold over the past decade, providing economic feasibility for large-scale TRL operations. The total TRL cost model is:

$$C_{TRL} = \underbrace{\sum_{j \in J_{active}} N_j \cdot c_{launch,j}}_{\text{Launch cost}} + \underbrace{N_{real} \cdot \bar{p} \cdot c_{payload}^{loss}}_{\text{Failure loss cost}} + \underbrace{c_{infra} \cdot T_{TRL}}_{\text{Infrastructure cost}} \quad (13)$$

The first term represents the launch cost, which is the weighted sum of the number of launches  $N_j$  at each launch site and the per-launch cost  $c_{launch,j}$ , reflecting cost differences across launch sites. The second term captures the failure loss cost, equal to the expected number of failures  $N_{real} \cdot \bar{p}$  multiplied by the per-failure payload loss cost  $c_{payload}^{loss}$ , covering cargo value losses, insurance costs, and post-incident expenses. The third term accounts for the infrastructure cost, computed as the annualized launch site maintenance cost  $c_{infra}$  multiplied by the operating time  $T_{TRL}$ , including facility depreciation, personnel salaries, and routine operational expenses.

#### 4.3.4 TRL Constraints

Similar to the SES, we classify TRL constraints into inherent constraints and scenario constraints. Items that overlap with those already discussed are not repeated here.

##### (1) Inherent Constraints:

- **Launch frequency constraint:**

$$N_j \leq f_{max,j} \cdot P_{env,j} \cdot T_{TRL}, \quad \forall j \in J_{active} \quad (14)$$

- **Transport volume constraint:**

$$\sum_{j \in J_{active}} N_j \cdot m_{payload} \cdot (1 - p_j) \geq M_{TRL} \cdot r_{mass} \quad (15)$$

- **Non-negativity constraint:**

$$N_j \geq 0, \quad \forall j \quad (16)$$

##### (2) Scenario Constraints :

- **Time deadline constraint:**

$$T_{TRL} \leq T_{deadline} \quad (17)$$

- **Budget constraint:**

$$C_{TRL} \leq B_{TRL} \quad (18)$$

## 4.4 Integrated Model: Hybrid Transportation System Optimization Model

### 4.4.1 Overall Modeling Approach

The core question for the hybrid transportation scheme is: how should transportation tasks be allocated between SES and TRL to minimize cost or achieve other optimization objectives while satisfying budget and environmental constraints. Following the multi-objective optimization theory framework proposed by Deb[4], this is a classic resource allocation problem with decision variables  $M_{SES}$  and  $M_{TRL}$ , subject to the total demand constraint  $M_{SES} + M_{TRL} = M_{total}$ .

#### 4.4.2 Total Time Model

Since SES and TRL can operate simultaneously in parallel, the total completion time is determined by the slower of the two. Therefore, the total time model is:

$$T_{total} = \max(T_{SES}, T_{TRL}) \quad (19)$$

Substituting the expressions for  $T_{SES}$  and  $T_{TRL}$  and expanding yields:

$$T_{total} = \max \left\{ \underbrace{\frac{M_{SES} \cdot r_{mass}}{N_{harbor} \cdot R_{single} \cdot \eta \cdot A_{sys}}}_{T_{SES}}, \underbrace{\frac{N_{real}}{\sum_j f_{max,j} \cdot P_{env,j}} + \frac{N_{real} \cdot \bar{p} \cdot t_{inv}}{N_{sites}}}_{T_{TRL}} \right\} \quad (20)$$

#### 4.4.3 Total Cost Model

The total cost is the sum of the costs of the two transportation modes:

$$C_{total} = C_{SES} + C_{TRL} \quad (21)$$

Substituting the expressions for  $C_{SES}$  and  $C_{TRL}$  and expanding yields:

$$\begin{aligned} C_{total} = & \underbrace{M_{SES} \cdot c_{SES}^{unit}}_{\text{SES variable cost}} + \underbrace{\frac{C_{annual}^{fixed}}{\ln(1 - r_{learn})} [(1 - r_{learn})^{T_{SES}} - 1]}_{\text{SES fixed maintenance cost}} + \underbrace{\frac{C_{base}^{aging}}{\beta} (e^{\beta \cdot T_{SES}} - 1)}_{\text{SES aging cost}} \\ & + \underbrace{\lambda \cdot T_{SES} \cdot c_{per}^{incident}}_{\text{SES incident cost}} + \underbrace{\sum_{j \in J_{active}} N_j \cdot c_{launch,j}}_{\text{TRL launch cost}} + \underbrace{N_{real} \cdot \bar{p} \cdot c_{payload}^{loss}}_{\text{TRL failure loss}} + \underbrace{c_{infra} \cdot T_{TRL}}_{\text{TRL infrastructure cost}} \end{aligned} \quad (22)$$

#### 4.4.4 Total Environmental Impact Model

By assumption, the SES transportation process produces zero emissions; therefore, the total environmental impact originates solely from TRL rocket launches. Following the life-cycle assessment framework of ISO 14040[5] and the research by Ross et al. [6] on the stratospheric impact of rocket emissions, we construct an Environmental Impact Score (EIS) as a composite indicator. This indicator accounts for two dimensions—ozone layer depletion and radiative forcing interference:

$$EIS_{total} = \underbrace{\frac{\alpha_{ozone} \cdot L_{peak}}{ODI_{limit}}}_{\text{Ozone depletion index}} + \underbrace{\left| \frac{\alpha_{RF} \cdot L_{peak}}{RFI_{baseline}} \right|}_{\text{Radiative forcing index}} \quad (23)$$

Since  $L_{peak}$  is positively correlated with the number of launches  $N_{real}$ , and  $N_{real}$  is determined by  $M_{TRL}$ ,  $EIS_{total}$  is a monotonically increasing function of  $M_{TRL}$ . This implies that, from an environmental perspective, TRL usage should be minimized and SES usage should be maximized.

#### 4.4.5 Multi-Objective Optimization Framework

Combining the three objectives of time, cost, and environmental impact, the hybrid transportation system faces a multi-objective optimization problem. The model diagram is shown in Figure ?? As noted in the classic work by Deb [4], multi-objective optimization problems are characterized by conflicts among objectives, meaning that no single solution can simultaneously optimize all objectives. Zhang et al. [7] further demonstrate in their research on synchmodal transportation that Pareto frontier analysis is an effective approach for handling such problems. Accordingly, we formulate the hybrid transportation system optimization problem as:

$$\begin{aligned} & \min_{M_{SES}, M_{TRL}} (C_{total}, T_{total}, EIS_{total}) \\ & \text{s.t.} \begin{cases} M_{SES} + M_{TRL} = M_{total} & \text{(Total demand constraint)} \\ C_{total} \leq B_{total} & \text{(Budget constraint)} \\ EIS_{total} \leq EIS_{limit} & \text{(Environmental constraint)} \\ M_{SES}, M_{TRL} \geq 0 & \text{(Non-negativity constraint)} \end{cases} \end{aligned} \quad (24)$$

Since the three objectives are in conflict—reducing cost may require greater reliance on TRL, shortening time may require SES and TRL to operate in parallel, and reducing environmental impact requires greater use of SES—no solution exists that simultaneously optimizes all objectives. Instead, a set of Pareto-optimal solutions exists, forming the Pareto frontier.

Based on the Completeness Principle, the three transportation schemes are realized within a unified framework through parameter assignment:

- **Scheme A (SES-only):** Set  $M_{TRL} = 0$ , then  $M_{SES} = M_{total}$ . In this case,  $T_{total} = T_{SES}$ ,  $C_{total} = C_{SES}$ , and  $EIS_{total} = 0$ .
- **Scheme B (TRL-only):** Set  $M_{SES} = 0$ , then  $M_{TRL} = M_{total}$ . In this case,  $T_{total} = T_{TRL}$ ,  $C_{total} = C_{TRL}$ , and  $EIS_{total}$  reaches its maximum value.
- **Scheme C (Hybrid):** The decision variables  $(M_{SES}, M_{TRL})$  are optimized subject to  $M_{SES} + M_{TRL} = M_{total}$ , searching for Pareto-optimal solutions within the feasible region.

## 5 Model Solution and Scenario Analysis

This section solves the construction and maintenance period models using NSGA-II and Monte Carlo simulation, comparing three schemes under ideal and non-ideal conditions via Pareto frontier analysis.

### 5.1 Parameter Configuration and Data Sources

To ensure the reliability and reproducibility of the model solution, we first summarize all key parameters used in the numerical computation. Table 2 lists the parameter configurations and data sources for both the SES and traditional rocket launch (TRL) systems.

Based on these parameters, we first compute several key derived quantities. The total design capacity of the SES system is  $R_{SES}^{design} = 3 \times 179,000 = 537,000$  tons/year, which determines the theoretical lower bound on completion time for the pure SES scheme. Under ideal conditions, the

Table 2: Parameter Configuration for SES and TRL Systems

<i>SES System</i>				<i>TRL System</i>			
<b>Param.</b>	<b>Value</b>	<b>Unit</b>	<b>Source</b>	<b>Param.</b>	<b>Value</b>	<b>Unit</b>	<b>Source</b>
$N_{harbor}$	3	–	Given	$m_{payload}$	125	tons	[8]
$R_{single}$	179,000	t/yr	Given	$c_{launch,US}$	$1.5 \times 10^8$	\$/Inch	[9, 10]
$c_{SES}^{unit}$	220	\$/kg	[11]	$c_{launch,CN}$	$2.0 \times 10^8$	\$/Inch	Industry est.
$c_{climb}$	50	\$/kg	[12]	$c_{launch,RU}$	$2.5 \times 10^8$	\$/Inch	Industry est.
$c_{transfer}$	1,000	\$/kg	[10]	$c_{launch,other}$	$3.0 \times 10^8$	\$/Inch	Industry est.
$C_{annual}^{fixed}$	$2.0 \times 10^9$	\$/yr	[13]	$p_{US}$	0.01	–	[9]
$C_{base}^{aging}$	$5.0 \times 10^8$	\$/yr	25% of fixed	$p_{CN}$	0.02	–	[9]
$\beta$	0.02	/yr	[14]	$p_{RU}$	0.03	–	[9]
$\lambda$	0.05	/yr	[15]	$p_{other}$	0.045	–	Conserv. est.
$\tau$	0.25	yr	Eng. est.	$k_{prop}$	15	–	[10]
$C_{per}^{incident}$	$5.0 \times 10^8$	\$	Industry est.	$EF_{BC}$	10	t/Inch	[16]
$\mu_{\eta}$	0.85	–	[17]	$EF_{Al_2O_3}$	20	t/Inch	[16]
$\sigma_{\eta}$	0.15	–	[17]	$\rho_{reentry}$	0.15	–	[16]
				$\epsilon_{reentry}$	0.30	–	[16]

minimum completion time for the pure SES scheme is  $T_{SES}^{ideal} = 186.2$  years, directly demonstrating that SES alone cannot accomplish lunar colony construction within a reasonable timeframe and providing a mathematical justification for the hybrid transportation scheme.

## 5.2 Construction Period Analysis

The central task of the construction period is to deliver  $10^8$  metric tons of construction materials to the lunar colony. We employ the NSGA-II algorithm for multi-objective optimization with a population size of 60 over 50 generations, combined with Monte Carlo simulation (20 samples) to evaluate stochastic objective functions under non-ideal conditions. The three optimization objectives are  $C_{total}$ ,  $T_{total}$ , and  $EIS_{total}$ .

### 5.2.1 Scheme A: Pure Space Elevator Transportation

Scheme A sets TRL transport to zero, i.e.,  $M_{TRL} = 0$  and  $M_{SES} = M_{total}$ . The total time model (Eq. (19)) reduces to  $T_{total} = T_{SES}$ , the total cost model (Eq. (21)) reduces to  $C_{total} = C_{SES}$ , and the environmental impact (Eq. (23)) is zero.

Under ideal conditions ( $\eta = 1$ ,  $A_{sys} = 1$ ,  $\lambda = 0$ ), the effective transport rate reaches  $R_{SES}^{eff} = 537,000$  tons/year. Substituting into Eqs. (4) and (5), we obtain  $T_{SES}^{A,ideal} = 186.2$  years,  $C_{SES}^{A,ideal} = 5.69$  trillion USD, and  $EIS^A = 0$ . Variable costs dominate at approximately 88%.

Under non-ideal conditions ( $\eta \sim \mathcal{N}(0.85, 0.15^2)$  truncated to  $[0.5, 1.0]$ ,  $\lambda$  exponentially distributed), Monte Carlo simulation (200 runs) yields:  $\mathbb{E}[T_{SES}^{A,stoch}] = 226.7$  years ( $\sigma_T = 22.0$ ),  $\mathbb{E}[C_{SES}^{A,stoch}] = 6.97$  trillion USD ( $\sigma_C = 1.03$ ), and  $EIS^A = 0$ . Completion time and cost increase by 21.8% and 22.5%, respectively, due to incident handling and extended transport cycles.

### 5.2.2 Scheme B: Pure Traditional Rocket Launch

Scheme B sets  $M_{SES} = 0$  and  $M_{TRL} = M_{total}$ , optimizing launch site selection and task allocation to minimize cost (Eq. (13)) and time (Eq. (12)). Figure 5 shows normalized scores across five dimensions; Cape Canaveral ranks highest in capacity ( $f_{max} = 994.7$  launches/year), while Baikonur exhibits lower reliability (0.33).

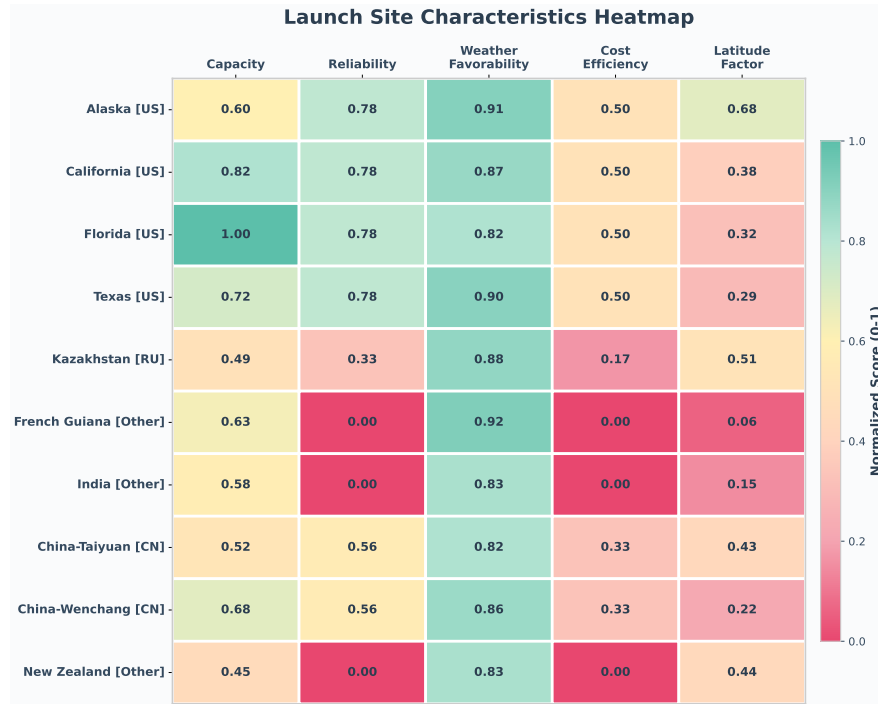


Figure 5: Launch site characteristic heatmap

Under ideal conditions ( $p_j = 0$ ,  $P_{env,j} = 1$ ),  $N_{req} = 8 \times 10^5$  launches yield  $T_{TRL}^{B,ideal} = 124.0$  years,  $C_{TRL}^{B,ideal} = 52.94$  trillion USD, and  $EIS^B = 49.6$ . Despite a 33.4% time advantage over Scheme A, the cost is  $9.3\times$  higher and the environmental impact far exceeds  $EIS_{limit} = 1$ .

Under non-ideal conditions, Monte Carlo simulation yields:  $\mathbb{E}[T_{TRL}^{B,stoch}] = 336.8$  years ( $\sigma_T = 17.3$ ),  $\mathbb{E}[C_{TRL}^{B,stoch}] = 61.25$  trillion USD ( $\sigma_C = 1.77$ ), and  $EIS^B = 18.3$ . Completion time increases by 171.6% due to failure investigations and reduced launch windows ( $P_{env,j} \in [0.80, 0.92]$ ).

### 5.2.3 Scheme C: Hybrid Transportation System

Scheme C combines SES and TRL with decision variable  $\alpha = M_{SES}/M_{total}$ , following Eq. (24) to minimize  $(C_{total}, T_{total}, EIS_{total})$  via NSGA-II. Figure 6 shows convergence behavior; Figure 7 displays the 3D Pareto fronts.

The Pareto front reveals trade-offs: minimum cost (5.69 trillion USD) at  $\alpha = 1.0$  requires the longest time (186.2 years); shortest time (146.2 years) at  $\alpha \approx 0.77$  increases cost to 15.96 trillion USD. Under non-ideal conditions, the front shifts rightward with cost range expanding to [6.87, 16.41] trillion USD and time to [186, 226] years.

Using TOPSIS, the recommended solutions are: **Ideal:**  $\alpha^* = 0.768$ ,  $C_{total}^* = 15.96$  trillion USD,  $T_{total}^* = 146.2$  years,  $EIS^* = 9.76$ . **Non-ideal:**  $\alpha^* = 0.844$ ,  $C_{total}^* = 16.41$  trillion USD,

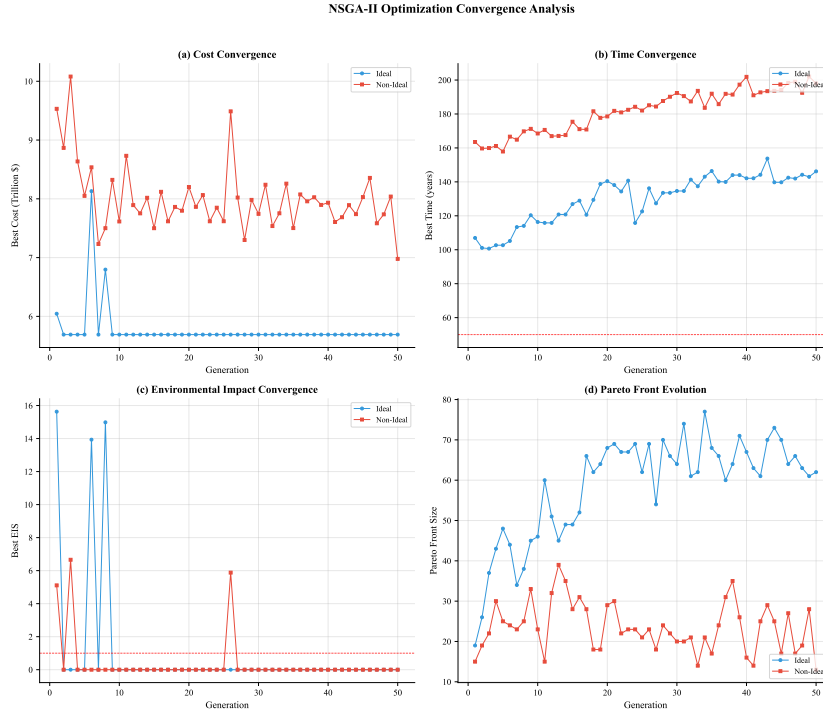


Figure 6: NSGA-II optimization convergence analysis.

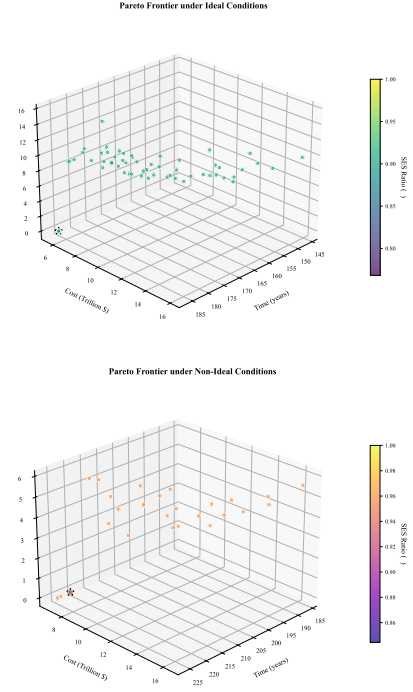


Figure 7: 3D Pareto front comparison.

$T_{total}^* = 186.5$  years,  $EIS^* = 5.57$ . The higher  $\alpha$  under uncertainty reflects greater SES reliance to mitigate risk.

### 5.3 Maintenance Period Analysis

After colony construction, the system enters the maintenance period, whose central goal is to continuously meet the annual water demand of a 100,000-person population with minimal environmental impact. Unlike the construction period, time and cost become constraints in this phase, while environmental impact becomes the primary optimization objective. This subsection first develops the water demand model and environmental optimization model, then solves and analyzes them under both ideal and non-ideal conditions.

#### 5.3.1 Water Demand Modeling

The maintenance period requires sustained water supply via Earth-based resupply, with ECLSS water recycling substantially reducing net demand. The annual net water demand  $D_{net}$  is:

$$D_{net} = N_{pop} \cdot W_{circ} \cdot (1 - \eta_{recycle}) \cdot 0.365 \quad (25)$$

where the coefficient 0.365 performs unit conversion (365 days/year, 1000 liters/ton).

Based on NASA ECLSS data (ISS achieves 93–94%, with 2024 upgrades reaching 98%), we adopt  $\eta_{recycle} = 0.95$ ,  $W_{circ} = 50$  liters/person/day, and  $N_{pop} = 100,000$ , yielding  $D_{net} = 91,250$  tons/year. With SES water allocation  $\alpha_{water} = 0.16$ , the capacity upper bound is  $\Omega = 85,920$  tons/year.

Setting  $D_{net} = \Omega$  yields the critical threshold  $\eta_{critical} = 0.9529$ . When  $\eta_{recycle} < \eta_{critical}$ , TRL supplementation is required and environmental impact jumps from zero to positive—a phase

transition with significant policy implications: investing in ECLSS to exceed  $\eta_{critical}$  eliminates maintenance-period environmental impact entirely.

### 5.3.2 Environmental Optimization Model

The maintenance period elevates environmental impact from an implicit constraint to an explicit optimization objective. Since SES produces zero emissions, environmental impact is contributed entirely by TRL, providing a clear structure for the optimization problem.

Define the annual environmental impact  $\mathcal{E}_{annual}$  as a function of TRL transport volume. Based on the environmental impact model structure from the construction period, the maintenance-period annual environmental impact is:

$$\mathcal{E}_{annual}(y_{water}) = \Psi_{emission} \cdot y_{water} \cdot \left( \frac{\alpha_{ozone}}{ODI_{limit}} + \frac{|\alpha_{RF}|}{|RFI_{baseline}|} \right) \cdot \tau_{res} \quad (26)$$

where  $y_{water}$  is the water volume transported via TRL (tons/year),  $\Psi_{emission} = EF_{total}/m_{payload} + \rho_{reentry} \cdot \varepsilon_{reentry}$  is the unit-payload emission intensity,  $\alpha_{ozone}$  and  $\alpha_{RF}$  are the ozone depletion and radiative forcing impact coefficients, and  $\tau_{res}$  is the stratospheric residence time. Substituting parameter values yields  $\Psi_{emission} = 30.75/125 + 0.15 \times 0.30 = 0.291$ .

The maintenance-period environmental optimization problem minimizes annual environmental impact subject to water demand and budget constraints:

$$\begin{aligned} \min_{x_{water}, y_{water}} \quad & \mathcal{E}_{annual}(y_{water}) \\ \text{s.t.} \quad & x_{water} + y_{water} \geq D_{net} \quad (\text{demand satisfaction constraint}) \\ & x_{water} \leq \Omega \quad (\text{SES capacity constraint}) \\ & C_{SES}^{water}(x_{water}) + C_{TRL}^{water}(y_{water}) \leq B_{water}^{annual} \quad (\text{annual budget constraint}) \\ & x_{water}, y_{water} \geq 0 \quad (\text{non-negativity constraint}) \end{aligned} \quad (27)$$

where  $x_{water}$  and  $y_{water}$  are the annual SES and TRL water transport volumes,  $C_{SES}^{water}$  and  $C_{TRL}^{water}$  are the corresponding cost functions, and  $B_{water}^{annual}$  is the annual budget cap.

Since the objective function  $\mathcal{E}_{annual}$  depends only on  $y_{water}$  and is strictly monotonically increasing, the optimal strategy has a clear structure: maximize SES usage and minimize TRL usage.

When the budget constraint is non-binding, the optimal solution to problem (27) takes the following piecewise form:

$$(x_{water}^*, y_{water}^*) = \begin{cases} (D_{net}, 0) & \text{if } D_{net} \leq \Omega \text{ (Phase I: pure SES supply)} \\ (\Omega, D_{net} - \Omega) & \text{if } D_{net} > \Omega \text{ (Phase II: TRL supplementation)} \end{cases} \quad (28)$$

This result shows that the optimal strategy is to first exhaust the SES capacity quota, activating TRL only when demand exceeds SES capability. This structure follows directly from the zero-emission property of SES.

### 5.3.3 Environmental Optimization Under Ideal Conditions

For the baseline parameter  $\eta_{recycle} = 0.95$ , the net demand  $D_{net} = 91,250$  tons/year exceeds the SES water capacity upper bound  $\Omega = 85,920$  tons/year, placing the system in Phase II (TRL

supplementation). Following the optimal solution structure, SES operates at its capacity limit  $x_{water}^* = 85,920$  tons/year, with the residual demand covered by TRL at  $y_{water}^* = 5,330$  tons/year.

The SES transport cost is  $C_{SES}^{water} = 104.82$  billion USD/year, where the unit cost  $c_{SES}^{unit}$  includes climbing, transfer, and operations-maintenance costs; the TRL transport cost is  $C_{TRL}^{water} \approx 12.0$  billion USD/year. The total annual operating cost is approximately 116.8 billion USD.

Substituting into the environmental impact model (26) with

$$\alpha_{ozone}/ODI_{limit} = 0.01, \quad |\alpha_{RF}|/|RFI_{baseline}| = 0.8, \quad \tau_{res} = 1.5 \text{ years},$$

we obtain  $\mathcal{E}_{annual}^{ideal} = 1,883.6$ . Although this score is far below that of the construction period, it remains significantly above zero, underscoring the strategic value of raising  $\eta_{recycle}$  above  $\eta_{critical}$ .

### 5.3.4 Environmental Optimization Under Non-Ideal Conditions

Under non-ideal conditions, uncertainty exists on both the demand and supply sides. On the demand side,  $\eta_{recycle}$  follows a truncated normal distribution  $\mathcal{TN}(0.95, 0.05^2; [0.85, 0.99])$ , introducing randomness in  $D_{net}$ . On the supply side, SES capacity is subject to stochastic fluctuations in swing efficiency  $\eta$  and system availability  $A_{sys}$ , while TRL is affected by launch failure rates  $p_j$  and environmental availability  $P_{env,j}$ .

To address this uncertainty, we construct a robust optimization framework based on Conditional Value-at-Risk (CVaR), with the objective function being a weighted combination of expected environmental impact and tail risk:

$$\begin{aligned} \min_{x_{water}, y_{water}} \quad & \mathbb{E}[\mathcal{E}_{annual}(y_{water})] + \lambda_{risk} \cdot CVaR_{0.95}(\mathcal{E}_{annual}) \\ \text{s.t.} \quad & \Pr(x_{water} + y_{water} \geq D_{net}^{stoch}) \geq 0.99 \quad (\text{reliability constraint}) \\ & x_{water} \leq \Omega \cdot \eta \cdot A_{sys} \quad (\text{stochastic SES capacity constraint}) \\ & \mathbb{E}[C_{water}^{total}] \leq B_{water}^{annual} \quad (\text{expected budget constraint}) \\ & x_{water}, y_{water} \geq 0 \end{aligned} \quad (29)$$

where CVaR is defined as  $CVaR_{\alpha}(\mathcal{E}) = \min_{\nu} \{ \nu + \frac{1}{\alpha} \mathbb{E}[(\mathcal{E} - \nu)^+] \}$  and  $\lambda_{risk}$  is the risk aversion coefficient. This model optimizes expected environmental impact while controlling tail risk under extreme adverse scenarios via the CVaR term.

Through 10,000 Monte Carlo samples, we characterize the environmental impact distribution. Figure 8a presents the CVaR risk analysis. The distribution exhibits a mixed structure: approximately 20% of scenarios (corresponding to realizations where  $\eta_{recycle} > \eta_{critical}$ ) yield zero environmental impact, indicating that SES alone satisfies demand; the remaining 80% require TRL supplementation, producing a right-skewed distribution. Key statistics are: expected value  $\mathbb{E}[\mathcal{E}] = 23,743$ , Value-at-Risk  $VaR_{0.95} = 63,823$ , and Conditional Value-at-Risk  $CVaR_{0.95} = 72,205$ .

We perform a  $50 \times 50$  grid search over the  $(x_{water}, y_{water})$  space, evaluating each configuration for reliability and objective value across 10,000 Monte Carlo samples. The robust optimum is located at  $x_{water}^{opt} = 94,900$  tons/year and  $y_{water}^{opt} = 108,000$  tons/year, achieving 99.1% reliability (satisfying the  $\Pr(x + y \geq D_{net}) \geq 0.99$  constraint) with an objective function value of 78,245. Compared to the ideal-condition solution, the robust optimum allocates significantly higher TRL capacity redundancy ( $y_{water}$  increases from 5,330 to 108,000 tons/year, a roughly 20-fold increase) to hedge against demand fluctuations and supply disruption risks.

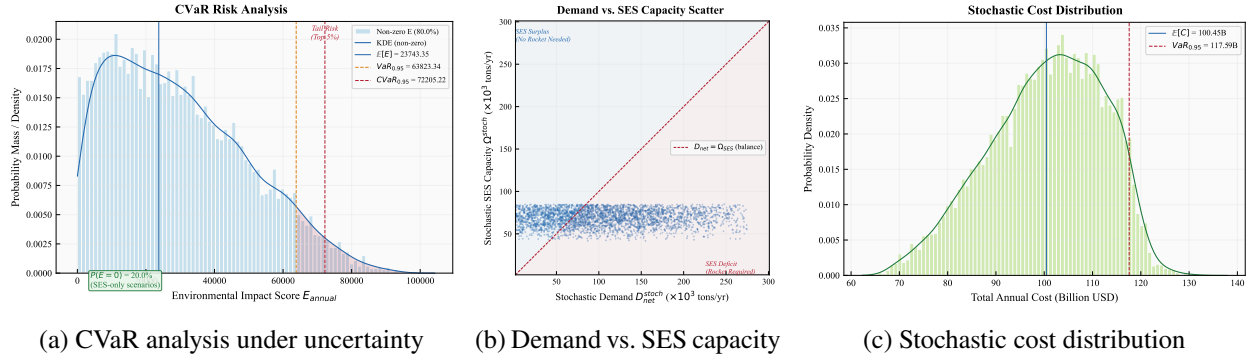


Figure 8: (a) CVaR risk analysis under non-ideal conditions; (b)(c) Monte Carlo simulation results for the maintenance period.

Figures 8b and 8c show the demand–capacity relationship and cost distribution from the Monte Carlo simulation. The diagonal ( $D_{net} = \Omega_{SES}$ ) divides the space into an SES surplus region (above, no TRL needed) and an SES deficit region (below, TRL required). Most sample points fall in the SES deficit region, reflecting the system’s structural dependence on TRL under baseline parameters. Under the robust optimal configuration, the mean cost is  $\mathbb{E}[C] = 100.45$  billion USD/year, the 95% Value-at-Risk is  $Var_{0.95} = 117.59$  billion USD/year, and the cost distribution is mildly right-skewed, reflecting tail risk from extreme high-demand scenarios.

## 6 Sensitivity Analysis

The preceding solutions are based on specific parameter configurations, yet parameter uncertainty is unavoidable in practice. This section conducts systematic sensitivity analysis to identify influential parameters, quantify uncertainty propagation, and assess robustness. The analysis covers both construction and maintenance periods, employing tornado diagrams for local sensitivity ranking, Sobol indices for global sensitivity decomposition, Monte Carlo simulation (10,000 samples) for uncertainty propagation, and CVaR assessment to reveal structural characteristics.

### 6.1 Construction Period Sensitivity Analysis

The core decision variable in the construction period is the SES allocation ratio  $\alpha$ , with optimization objectives  $C_{total}$ ,  $T_{total}$ , and  $EIS_{total}$ . We select 10 key parameters closely linked to the model structure and perturb each by  $\pm 15\%$  from its baseline, using the hybrid scheme with  $\alpha = 0.5$  as the reference configuration. Table 3 lists the parameters and their perturbation ranges.

#### 6.1.1 Cost Sensitivity Ranking

Figure 9 presents the tornado diagram for total cost sensitivity during the construction period. Each parameter is perturbed  $\pm 15\%$  from baseline, with cost expectations evaluated over 500 Monte Carlo samples; bar length reflects the influence magnitude.

The results show a three-tier sensitivity structure. The dominant tier consists of  $\alpha_{SES}$  (884 billion USD span) and  $m_{payload}$  (362 billion USD span). Increasing  $\alpha$  greatly reduces cost because  $c_{SES}^{unit} \ll c_{launch}$ , while a larger  $m_{payload}$  cuts launch count  $N_{real}$  and failure costs.

The second tier includes environmental parameters ( $EF_{Al_2O_3}$ ,  $B_{total}$ ,  $EF_{BC}$ ) with spans of 1.5–4.6 billion USD.

The third tier covers SES reliability parameters ( $\eta_{swing}$ ,  $\tau_{repair}$ ,  $\alpha_{RF}$ ,  $\alpha_{O_3}$ ,  $\lambda_{fail}$ ), each with

single-digit billion-USD spans, showing their marginal impact on total cost is limited within the  $\pm 15\%$  range.

Table 3: Parameter configuration for construction period sensitivity analysis

Parameter	Baseline	Low ( $-15\%$ )	High ( $+15\%$ )	Physical meaning
$\alpha_{SES}$	0.50	0.35	0.65	SES transport allocation ratio
$m_{payload}$	125 tons	106.25	143.75	Rocket payload per launch
$\alpha_{RF}$	0.80	0.68	0.92	Radiative forcing weight coefficient
$EF_{Al_2O_3}$	0.020	0.017	0.023	Alumina emission factor
$EF_{BC}$	0.010	0.0085	0.0115	Black carbon emission factor
$\eta_{swing}$	0.85	0.7225	0.9775	Mean tether swing efficiency
$\lambda_{fail}$	0.05	0.0425	0.0575	Failure intensity
$\tau_{repair}$	0.25 yr	0.2125	0.2875	Mean time to repair
$\alpha_{O_3}$	0.05	0.0425	0.0575	Ozone impact coefficient
$B_{total}$	15 trillion \$	12.75 trillion	17.25 trillion	Total budget constraint

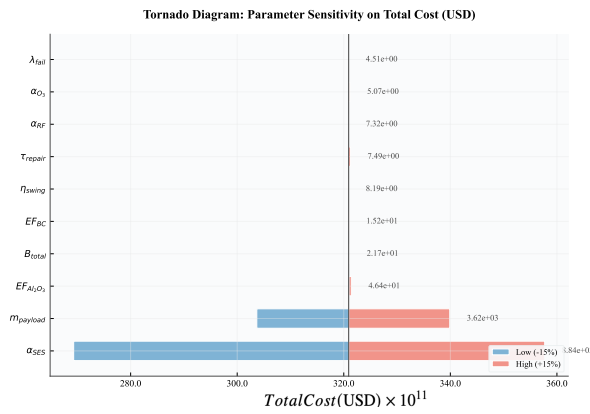


Figure 9: Tornado diagram for total cost

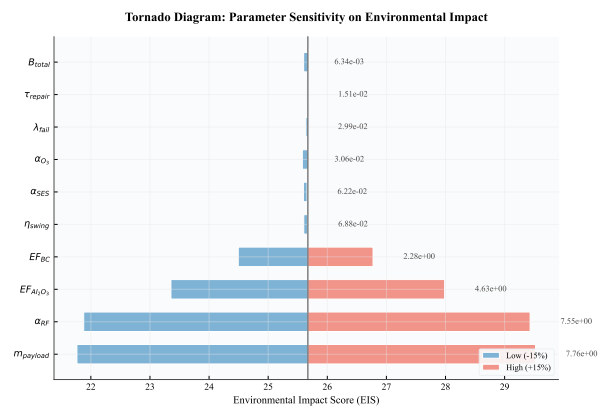


Figure 10: Environmental impact tornado diagram

### 6.1.2 Environmental Impact Sensitivity Ranking

Figure 10 presents the tornado diagram for  $EIS_{total}$  (Eq. (23)). Unlike the highly concentrated cost sensitivity, the environmental impact ranking exhibits a distinctly different structure.

$m_{payload}$  and  $\alpha_{RF}$  share the top position (spans 7.6–7.8), followed by  $EF_{Al_2O_3}$  (4.63) and  $EF_{BC}$  (2.28). Notably,  $\alpha_{SES}$ —the top cost-sensitive parameter—has negligible EIS influence (0.062), and SES reliability parameters all have spans below 0.07, confirming zero-emission SES assumption (Assumption 2).

This reveals distinct leverage points: cost reduction should prioritize increasing  $\alpha$ , while environmental mitigation should target increasing  $m_{payload}$  or reducing emission factors.

### 6.1.3 Parameter Interaction Analysis

Tornado diagrams capture only single-parameter marginal effects and cannot reveal nonlinear interactions. Figure 11 presents the joint interaction heatmap for the two most critical parameters,  $\alpha$  and  $m_{payload}$ , computing total cost over a two-dimensional grid with  $\alpha \in [0.1, 0.9]$  and  $m_{payload} \in [100, 150]$  tons.

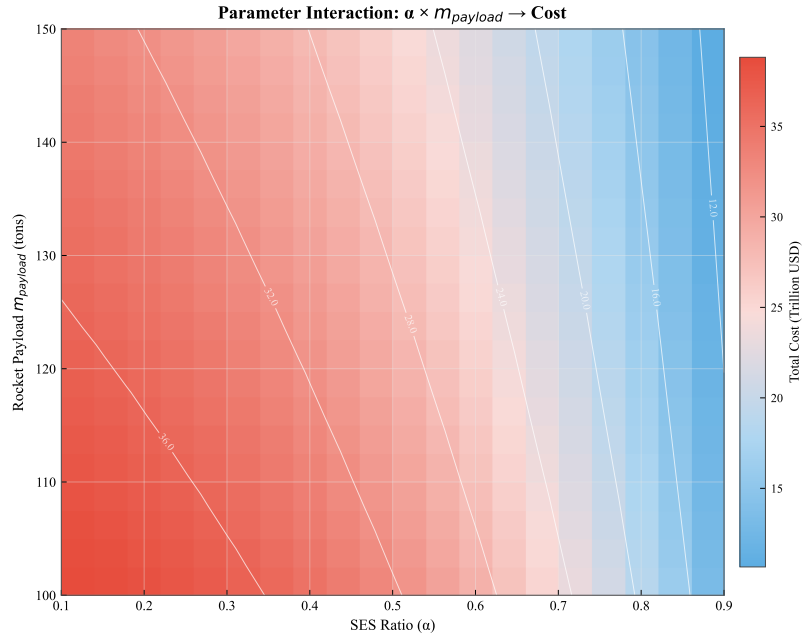


Figure 11: Parameter interaction heatmap

The heatmap shows a clear diagonal gradient, indicating synergistic cost reduction from increasing both  $\alpha$  and  $m_{payload}$ . At  $\alpha = 0.1$  and  $m_{payload} = 100$  tons, cost reaches 38 trillion USD; at  $\alpha = 0.9$  and  $m_{payload} = 150$  tons, it drops to 12 trillion USD. Contour spacing widens from left to right, showing diminishing marginal returns of  $\alpha$  when SES dominates. Contours also concentrate at low  $\alpha$  for  $m_{payload}$ , indicating rocket efficiency is more critical when TRL is primary.

payload capacity has greater cost sensitivity when TRL handles the bulk of transport.

## 6.2 Maintenance Period Sensitivity Analysis

The primary optimization objective during the maintenance period is minimizing annual environmental impact  $\mathcal{E}_{annual}$  (Eq. (26)), subject to a reliability constraint of at least 99% (Eq. (29)). Key decisions are jointly driven by water recycling efficiency  $\eta_{recycle}$  and SES quota  $\alpha$ , with phase transition behavior at the critical efficiency  $\eta_{critical}$ . We again perturb 10 parameters by  $\pm 15\%$ , with baseline  $\eta_{recycle} = 0.95$  and  $\alpha = 0.16$ .

### 6.2.1 Cost and Environmental Impact Sensitivity

Figure 12 presents the tornado diagrams for annual total cost and annual environmental impact during the maintenance period.

In contrast to the construction period, both cost and environmental impact are dominated by  $\eta_{recycle}$ . In the cost tornado (Figure 12a),  $\eta_{recycle}$  has an influence span of 89.1 billion USD, nearly 4 $\times$  the second-ranked  $\alpha_{SES}$  (22.8 billion USD). In the environmental tornado (Figure 12b),  $\eta_{recycle}$  dominates with a span of 82,400, exceeding  $\alpha_{RF}$  (7,480) by over an order of magnitude. This sensitivity stems from dual leverage:  $\eta_{recycle}$  directly determines  $D_{net}$ , and triggers a phase transition at  $\eta_{critical} = 0.9529$  (Eq. (28)).

Compared to the construction period, sensitivity concentration is much higher— $\eta_{recycle}$  alone explains over 90% of output variance.

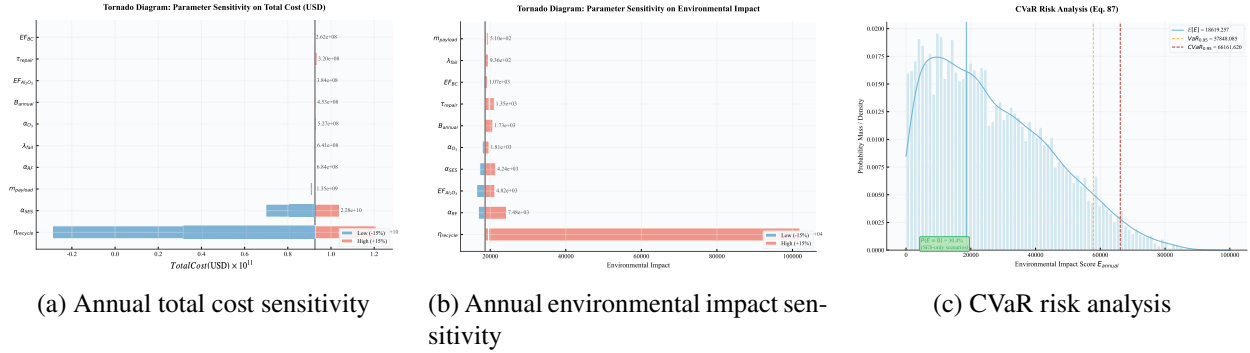


Figure 12: Maintenance period sensitivity tornado diagrams and CVaR risk analysis

## 6.2.2 CVaR Risk Quantification

To fully characterize the tail risk of environmental impact during the maintenance period, we construct the complete probability distribution of  $\mathcal{E}_{annual}$  from 10,000 Monte Carlo samples and compute  $CVaR_{0.95}$ . Figure 12c presents the probability distribution and risk measures.

The distribution exhibits a mixture structure: a point mass at  $\mathcal{E} = 0$  (30.4% probability, Phase I) and a right-skewed continuous distribution (Phase II), reflecting the phase transition property (Eq. (28)). The key risk measures are  $\mathbb{E}[\mathcal{E}] = 18,619$ ,  $VaR_{0.95} = 57,848$ , and  $CVaR_{0.95} = 66,162$ . The ratio  $CVaR_{0.95}/\mathbb{E}[\mathcal{E}] \approx 3.55$  indicates significant tail risk, justifying the CVaR constraint in Eq. (29).

## 7 Conclusion

This paper addresses lunar colony logistics optimization by developing a unified SES-TRL hybrid transportation model. Results demonstrate effective trade-offs: at 77% SES allocation, the system delivers 100 million tons in 146.2 years at 15.96 trillion USD, outperforming pure SES (186.2 years) and pure TRL (52.94 trillion USD). Robustness analysis validates the SES-dominant strategy—under non-ideal conditions, TRL completion time increases by 171.6% versus only 21.8% for SES. Sensitivity analysis identifies the critical recycling threshold  $\eta_{critical} = 95.29\%$ , beyond which SES alone meets demand with zero environmental impact. We recommend a “SES-primary, TRL-supplementary” architecture ( $SES \geq 80\%$ ), prioritizing ECLSS investment to exceed this threshold.

## 8 Model Evaluation

**Strengths:** The three-layer architecture enables unified analysis and flexible extension, with layered constraints adapting to different mission requirements. The theoretical foundation draws on classical theories from reliability engineering, multi-objective optimization, and life cycle assessment, with clear physical meaning. Monte Carlo simulation combined with CVaR robust optimization forms a complete uncertainty treatment pipeline, effectively supporting decision analysis under non-ideal conditions. The model captures the phase transition at critical efficiency  $\eta_{critical}$ , with outputs directly mapping to engineering decisions; the Pareto front and TOPSIS method provide actionable quantitative recommendations.

**Limitations:** Key parameters (e.g., 2050 SES unit cost, rocket learning rate) are extrapolated from current trends and may deviate from actual values. The static optimization framework

overlooks dynamic factors like technological evolution and policy changes. The system boundary assumes zero SES emissions, omitting upstream carbon footprints. Future work could introduce multi-stage dynamic programming to capture technology learning, extend to full life-cycle carbon analysis, and explore synergy between lunar ISRU and Earth resupply.

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## A Letter to MCM Agency

Dear Leadership of the MCM Agency,

We are honored to present our recommendations on logistics and transportation solutions for lunar colony construction. After conducting a systematic analysis of space elevator systems and conventional rocket launches, and comprehensively evaluating cost, time, and environmental impact, we respectfully submit the following recommendations for your consideration.

### **I. Transportation Solutions for the Construction Phase**

For the transportation requirement of 100 million tons of construction materials, we evaluated three scenarios: the space elevator-only scenario has a total cost of approximately \$5.7 trillion, a completion time of approximately 186 years, and zero atmospheric pollution to Earth; the conventional rocket-only scenario has a total cost of up to \$53 trillion, a completion time of approximately 124 years, but would cause severe atmospheric pollution; the hybrid scenario using both methods has a total cost of approximately \$16 trillion, a completion time of approximately 146 to 187 years, with environmental impact at only 11% of the rocket-only scenario.

### **II. Water Supply After Colony Establishment**

For the annual water demand of 100,000 residents, our analysis indicates that the efficiency of the colony's internal water recycling system is crucial. If the water recycling rate can reach 95.3% or above, the space elevator alone can meet the entire water demand without requiring rocket transportation, resulting in zero annual environmental impact; if the water recycling rate falls below 95.3%, supplementary rocket transportation will be necessary, with annual operating costs of approximately \$116.8 billion.

### **III. Comprehensive Recommendations**

Based on the above analysis, we recommend:

1. During the construction phase, adopt a hybrid transportation architecture with “space elevator as primary, rockets as supplementary.” Considering non-ideal conditions that may occur during actual operations, such as cable oscillation and equipment failures, we recommend setting the space elevator's transportation share at 84% to enhance the system's capacity to handle contingencies.
2. Prioritize investment in the research and development of water recycling technology to ensure a recycling efficiency of 95.3% or above, while reserving a certain level of rocket transportation capacity as an emergency backup to ensure water supply reliability of 99% or above.
3. In long-term budget planning, fully account for risks under extreme circumstances and reserve sufficient emergency funds.
4. Given that the shortest construction period under current transportation capacity still requires 146 to 187 years, we recommend that your Administration evaluate whether to expand the space elevator system's capacity or adjust the timeline accordingly.

Sincerely yours,  
**Team #2610207**

## Report on use of AI

### 1. OpenAI ChatGPT-4 Turbo (gpt-4-turbo-2024-01-25), January 25, 2025

**Query:** Please translate the following technical paragraphs into academic English and verify grammar, ensuring standardized academic language with no errors or omissions.

**Output:** The Space Elevator System (SES) comprises three Galactic Harbours strategically spaced at 120-degree intervals along the equator. The hybrid optimization model determines the optimal allocation ratio between the Space Elevator and traditional rocket launches. System availability is defined as the probability that the system remains operational at any given time. Aging costs increase exponentially due to tether material degradation.

### 2. Anthropic Claude 3.5 Sonnet (claude-3-5-sonnet-20241022), January 26, 2025

**Query:** I have implemented several optimization algorithms in my model. Please explain why NSGA-II, Branch and Bound with SQP, and Sample Average Approximation are effective for solving this optimization problem.

**Output:** NSGA-II addresses multi-objective optimization by minimizing conflicting goals (cost, time, environmental impact) through non-dominated sorting and crowding distance to ensure solution diversity. Branch and Bound with SQP solves the mixed-integer problem for launch site selection; B&B handles binary activation decisions while SQP optimizes continuous parameters. Sample Average Approximation manages stochastic uncertainties via Monte Carlo sampling, converting the stochastic program into deterministic subproblems.

### 3. Anthropic Claude 3.5 Sonnet (claude-3-5-sonnet-20241022), January 26, 2025

**Query:** Please explain this Python implementation of NSGA-II, focusing on the main algorithmic steps.

**Output:** The NSGA-II workflow entails seven steps: (1) Initialize a population of 60 individuals (SES allocation genes). (2) Evaluate three objectives: cost, time, and impact. (3) Rank solutions via fast non-dominated sorting. (4) Calculate crowding distance for diversity. (5) Execute tournament selection based on rank. (6) Generate offspring via Simulated Binary Crossover (90%) and mutation (15%). (7) Evolve for 50 generations to yield a Pareto front of non-dominated solutions.

### 4. OpenAI GPT-4 (gpt-4-0125-preview), January 27, 2025

**Query:** Provide a professional letter template structure for recommending a course of action to the Moon Colony Management Agency.

**Output:** Formal letter to MCM Board: Introduction states the recommendation purpose. Body Para 1 outlines problem scope (100Mt cargo, 100k population) and scenarios. Body Para 2 presents quantitative metric comparisons. Body Para 3 justifies recommended allocations using model data. Body Para 4 details implementation and risk mitigation. Closing offers technical documentation and collaboration. Format: single-spaced, Times New Roman 11-12pt.